P.13.

5. In each case, sketch the set of points determined by the given condition:

(a) $|z-1+i|=1$; (b) $|z+i| \le 3$; (c) $|z-4i| \ge 4$.

P.16.

7. Show that
$$|\mathrm{Re}\,(2+\bar{z}+z^3)| \le 4 \quad \text{when } |z| \le 1.$$

13. Show that the equation $|z-z_0|=R$ of a circle, centered at z_0 with radius R, can be written as
$$|z|^2 - 2\mathrm{Re}(z\bar{z}_0) + |z_0|^2 = R^2.$$

14. Using expressions (6), Section 6, for $\mathrm{Re}\,z$ and $\mathrm{Im}\,z$, show that the hyperbola $x^2-y^2=1$ can be written
$$z^2+\bar{z}^2=2.$$

$\mathrm{Re}\,z = \dfrac{z+\bar{z}}{2}$ and $\mathrm{Im}\,z = \dfrac{z-\bar{z}}{2i}$.

P.23.

1. Find the principal argument $\mathrm{Arg}\,z$ when

(a) $z = -\dfrac{2}{1+\sqrt{3}i}$; (b) $z=(\sqrt{3}-i)^6$.

P.31.

3. Find $(-8-8\sqrt{3}i)^{1/4}$, express the roots in rectangular coordinates, exhibit them as the vertices of a certain square, and point out which is the principal root.

6. Find the four zeros of the polynomial z^4+4, one of them being
$$z_0=\sqrt{2}e^{i\pi/4}=1+i.$$
Then use those zeros to factor z^2+4 into quadratic factors with real coefficients.

P.34. 1. Sketch the following sets and determine which are domains.

(a) $|z-2+i| \le 1$,　(b) $|2z+3|>4$;　(c) $\operatorname{Im} z > 1$;

(d) $\operatorname{Im} z = 1$,　(e) $0 \le \arg z \le \pi/4 \ (z \ne 0)$;　(f) $|z-4| \ge |z|$.

4. In each case, sketch the closure of the set:

(a) $-\pi < \arg z < \pi \ (z \ne 0)$;　(b) $|\operatorname{Re} z| < |z|$

(c) $\operatorname{Re}\left(\frac{1}{z}\right) \le \frac{1}{2}$;　(d) $\operatorname{Re}(z^2) > 0$.

P.61,

3. Using results in Sec. 20 show that

(a) a polynomial

$$P(z) = a_0 + a_1 z + a_2 z^2 + \cdots + a_n z^n \qquad (a_n \ne 0)$$

of degree n $(n \ge 1)$ is differentiable everywhere, with derivative

$$P'(z) = a_1 + 2a_2 z + \cdots + n a_n z^{n-1};$$

(b) the coefficients in the polynomial $P(z)$ in part (a) can be written

$$a_0 = P(0), \quad a_1 = \frac{P'(0)}{1!}, \quad a_2 = \frac{P''(0)}{2!}, \ \ldots, \ a_n = \frac{P^{(n)}(0)}{n!}.$$

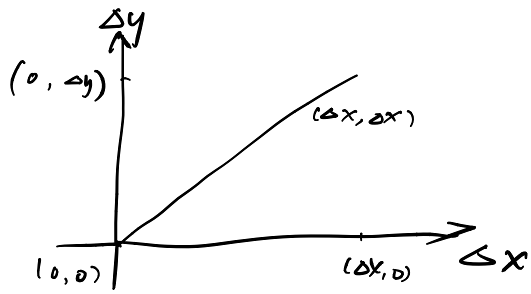
8. Use the method in Example 2, Sec 19, to show that $f'(z)$ does not exist at any point z when

(a) $f(z) = \operatorname{Re} z$;　(b) $f(z) = \operatorname{Im} z$.

9. Let f denote the function whose values are

$$f(z) = \begin{cases} \bar{z}^2/z & \text{when } z \ne 0 \\ 0 & \text{when } z = 0. \end{cases}$$

show that if $z=0$, then $\Delta w/\Delta z = 1$ at each non zero point on the real and imaginary axes in the Δz, or $\Delta x\, \Delta y$, plane. Then show that $\Delta w/\Delta z = -1$ at each non zero point $(\Delta x, \Delta y)$ on the line $\Delta y = \Delta x$ in the plane (Figure). Conclude from these observations that $f'(0)$ does not exist. Note that to obtain this result, it is not sufficient to consider only horizontal and vertical approaches to the origin in the Δz plane.



Figure